Influence of material and geometric parameters on the sensor based on active material

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To investigate the influence of the material properties and the geometric characteristics of a sensor based on active materials, the Least Angle Regression (LAR) method have been applied. The approximation function obtained from this uncertainty quantification method is used to undertake a global sensitivity analysis based on the Sobol indices. With the proposed method the most influential parameters can be determined.

***Index Terms*—Least Angle Regression, Piezoelectric, Sensitivity Analysis, Uncertainty Quantification.**

I. Introduction

In order to enhance the coupling effect, the sensors based on active materials such as piezoelectric, magnetostrictive or magnetoelectric operates at a resonance frequency [1], [2]. This operating frequency is usually fixed a priori at the mechanical resonance frequency [3]. Nevertheless, as the materials and the geometric properties present some uncertain characteristics due to manufacturing process deviations or lack of quality controls, the response of such sensors is not determinist. At an operating frequency determined a priori, the response of sensors depend on different factors. In [4], the Non Intrusive Approximation based on Least Angle Regression (LAR) [5] method was introduced. The method which enables to build a stochastic meta model based on an orthogonal polynomial basis, is closed to the original model of the sensor. This approach can be applied for the problem with high number of parameters [6]. The sensitivity analysis consisting in calculation the Sobol indices have been deduced directly from the meta model [7]. In this communication, this methodology is applied in the case of an active material sensor. The sensitivity analysis of the influence of the material properties and the geometric parameters on the sensor response at a operating frequency is done according to the methodology based on the LAR method.

II. Sensor based on active materials

The sensor based on active materials can be a piezoelectric or a magnetostrictive beam, or a combination of these two. To find the resonance frequency of the structure, one makes a frequency sweep and then determines the maximale response of the sensor on this frequency range. Figure 1 shows an example of a piezoelectric sensor made with a piezoelectric beam. the beam is clamped at the left border of the sensor. The sensor input is the electric voltage imposed on the up and down border of the beam. The output is the mechanical displacement at the up right extremity point of the sensor. The numerical model of the piezoelectric sensor is a 2D finite element model.

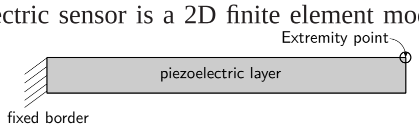

Fig. 1. Piezoelectric sensor

This piezoelectric sensor have uncertain material properties and geometric characteristics, leading to a dispersion of the sensor response which is the mechanical dipslacement at the extremity point shown in the Figure 1. In the stochastic approach, the uncertain parameters are modelled by random variables that we assume to be independent and uniformly distributed. We consider as random variables the length of the layer LL, the width of the layer LW, the elastic modulus E_{pzt}, the Poisson's ratio ν_{pzt}, the shear modulus G_{pzt}, the coupling coefficient d_{31} and d_33 of piezoelectric material. We consider the permittivity ϵ^x_{pzt} and ϵ^y_{pzt} following the x and y direction respectively, and the mass density ρ_{pzt}. The parameter variations have been assumed to be of 10% of nominal value for all parameters. In Tab I, we have reported the nominal value of each parameter.

	LL	LW	E_{pzt}	ν_{pzt}	G_{pzt}
m	0.005	3.2E-4	2.0E+9	0.29	7.75E+8

	d_{31}	d_{33}	ϵ^x_{pzt}	ϵ^y_{pzt}	ρ_{pzt}
m	9200	1.72E+6	15	3.85E+9	5.77E+9

TABLE I
Value and standard deviation of random variables

III. Sobol Sensitivity Analysis

The uncertainty quantification method is based on the Least Angle Regression. Let's consider a numerical model $\mathcal{M}(\boldsymbol{\xi})$ where $\boldsymbol{\xi}$ is a vector input of M random variables. These M random variables are independant and uniformly distributed in the interval [-1,1]. Using N realizations, the LAR method construct an approximation $\widetilde{Y}(\boldsymbol{\xi})$ of the numerical model $\mathcal{M}(\boldsymbol{\xi})$.

$$\widetilde{Y}(\boldsymbol{\xi}) = \sum_{i=1}^{P_{out}} \alpha_i \Psi_i(\boldsymbol{\xi}) \tag{1}$$

where $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_{P_{out}}\}$ the polynomial basis of P_{out} terms.

To make a global sensitivity analysis, several methods have been proposed. In this work, we choose the approach consisting of calculating the Sobol indices [8] because these indices can be deduced directly from (1). The approach consists of decomposition of the variance of the output as a sum of variances-based on single or mutual interaction of the uncertain inputs. This decomposition is given in the following form:

$$D = \sum_{1}^{N} D_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} D_{ij} + \ldots + D_{12\ldots n} \quad (2)$$

(2) leads to the calculation of the total variance of D as a sum of partial variance $D_{i_1,\ldots,i_s}$. The Sobol indices $S_{i_1,\ldots,i_s}$ is defined as:

$$S_{i_1,\ldots,i_s} = \frac{D_{i_1,\ldots,i_s}}{D} \quad (3)$$

Therefore we have the following property :

$$\sum_{1}^{N} S_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} S_{ij} + \ldots + S_{12\ldots n} = 1 \quad (4)$$

S_i where $i \in (1, \ldots, N)$ is the first order Sobol indice. It evaluates the effect of unique random variable u_i. The total Sobol index ST_i is calculated by:

$$ST_i = 1 - \frac{D_{\sim i}}{D} \quad (5)$$

where $D_{\sim i}$ is the variance which does not contain any effect corresponding to variable ξ_i. The bigger S_i is, the more influential ξ_i on the output. The smaller ST_i is, the less influential ξ_i on the output.

IV. Result

In the first analysis, the displacement following x and y direction are analyzed in a frequency range (from 0 to 15 kHz). The material properties and the geometric characteristics are at the nominal values. The result is presented in the Figure 2

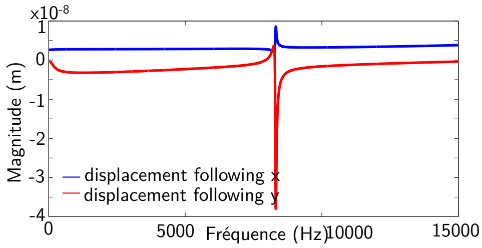

Fig. 2. Displacement versus frequency

Figure 2 shows that the deterministic model captures the resonance phenomenon. The resonance frequency is about 8.3kHz. Now we make the sensitivity analysis of 3 cases of operating frequency, the first case at a low frequency ($f_1 = 500Hz$), the second case at the resonance frequency ($f_2 = 8.3kHz$), and the third case at a frequancy slightly higher than the resonance frequency ($f_3 = 9kHz$). We have applied LAR method with 500 realizations regarding to 10 uncertain parameters. From the expansion obtained by (1), the first order and the total Sobol indices are calculated. Table II, III, IV gives the Sobol indices in 2 cases.

	LL	LW	E_{pzt}	ν_{pzt}	G_{pzt}
S_i	0.34	0.32	0	0	0
ST_i	0.34	0.32	0	0	0
	d_{31}	d_{33}	ϵ^x_{pzt}	ϵ^y_{pzt}	ρ_{pzt}
S_i	0.32	0	0	0	0
ST_i	0.32	0	0	0	0

TABLE II
Case 1: $f_1 = 500Hz$

	LL	LW	E_{pzt}	ν_{pzt}	G_{pzt}
S_i	0.10	0.06	0.00	0.00	0.00
ST_i	0.30	0.30	0.25	0.24	0.23
	d_{31}	d_{33}	ϵ^x_{pzt}	ϵ^y_{pzt}	ρ_{pzt}
S_i	0.05	0	0	0	0
ST_i	0.25	0.22	0.25	0.22	0.22

TABLE III
Case 1: $f_2 = 8.3kHz$

	LL	LW	E_{pzt}	ν_{pzt}	G_{pzt}
S_i	0.04	0.03	0.00	0.00	0.00
ST_i	0.34	0.23	0.30	0.28	0.20
	d_{31}	d_{33}	ϵ^x_{pzt}	ϵ^y_{pzt}	ρ_{pzt}
S_i	0.01	0	0	0	0
ST_i	0.27	0.19	0.27	0.24	0.33

TABLE IV
Case 1: $f_2 = 9.0kHz$

From Table II, the first-order and the total Sobol indices are very close. It means that there is no mutual interaction between the parameters. The most influential parameters are the length, the width of the sensor, and the coupling coefficient. In the second case, Table III shows the first-order Sobol indices are not as high as in the first case. The displacement does not depend mostly on a unique variable. The influence is the mutual interaction between two or several variables. Table IV shows the first-order Sobol indices are still reduced compared to the second case. The mutual interaction between the input is more and more important. It is shown that the mutual interaction has a great influence especially around the resonnant frequency. To determine which parameter interactions are the most influential, Sobol indices of higher order needs to be calculated. This study will be presented in the extended version.

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